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Linear and Nonlinear Analysis of a Nonconservative Frame of Divergence Instability

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Linear static, linear dynamic, and nonlinear static stability analyses are performed on a geometrically perfect two-member frame subjected to a compressive tangential load at its joint. On the basis of the linear analysis, the frame loses its stability through either a divergence or a flutter type of instability depending on the amount of joint stiffness. In the case of flutter-type instability, the static (linear or nonlinear) method of analysis cannot be employed. By using the nonlinear static analysis we establish that the frame loses its stability through a limit point. The nonlinear static analysis also sheds some light on the jump phenomenon in the critical load which is due to the application of the linear stability analysis. The displacements corresponding to that critical state are extremely large.

Introduction

PROBLEMS with the stability of nonconservative structural systems are frequently encountered in the modern design of aircraft and other structures. The equilibrium stability of discrete or continuous systems acted upon by nonconservative forces has been extensively investigated in recent years. However, only a few works on this topic are devoted to the postbuckling response of such systems.[1-4] Indeed, a review of the present state-of-the-art shows that the nonlinear response of nonconservatively loaded systems has not as yet been thoroughly investigated. The following are some of the pertinent questions needing clarification:

1) Is the failure of the static methods of analysis of the aforementioned systems due to the fact that nonlinear terms are not taken into account? In other words, is the static nonlinear stability analysis always applicable to pure flutter-type systems?

2) A linear analysis (either dynamic or static) leading to an eigenvalue problem does not give any information concerning the stability of the critical state. Are the magnitudes of the displacements corresponding to that state consistent with the limitations of linear stability theory? Are the types of critical points of conservative systems valid for nonconservative systems?

3) What is the nature of the jump phenomenon in the linear critical load of nonconservatively loaded systems occurring when the buckling mechanism of such systems changes from flutter to divergence instability and vice versa?

The main objective of the present investigation is to cast some light on these questions using simple methods of analysis. We chose the geometrically perfect frame model in Fig. 1, made from homogeneous, isotropic, and linearly elastic material with modulus of elasticity E. The quantities ℓ_i, A_i, I_i, and m_i $(i=1,2)$ are the length, constant cross-sectional area, constant moment of inertia, and mass per unit length, respectively, of each member. The frame is subjected at its joint to a constant magnitude follower compressive force P which remains tangent to the centerline of the vertical member during the deformation. Let w_i and ξ_i be the lateral and axial displacement of the centerline of the ith member with the sign convention shown in Fig. 1. In what follows, a linear (static and dynamic) analysis and a nonlinear static analysis are employed in discussing the actual response of the frame.

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Linear Stability Analysis

By resolving the follower force into a horizontal (nonconservative) component and a vertical (conservative) component, on the basis of the linear analysis we can make the following approximations

$$\beta^2 \sin w_1'(l) \simeq \beta^2 w_1'(l), \qquad \beta^2 \cos w_1'(l) \simeq \beta^2 \tag{1}$$

where $\beta^2 = P\ell_1^2/EI_1$ and $(\ )' = \mathrm{d}(\ )/\mathrm{d}x$. Subsequently, the classical static and dynamic methods of stability analysis are employed.

Static Analysis

The following differential equations governing the equilibrium of the frame are valid

$$w_1'''' + \beta^2 w_1'' = 0, \qquad w_2'''' = 0 \tag{2}$$

Integration of these equations and use of the support conditions result in

$$w_1(x_1) = A(\sin\beta x_1 - \beta x_1) + B(\cos\beta x_1 - 1)$$
$$w_2(x_2) = \bar{A}x_2^3 + \bar{\Gamma}x_2 \tag{3}$$

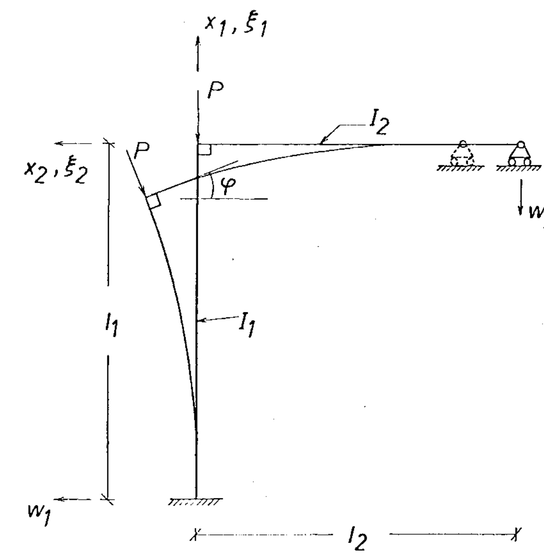

Fig. 1 Geometry and sign convention.

Application of the boundary conditions

$$w_1'''(1)=0, \quad w_2(1)=0, \quad w_1'(1)-w_2'(1)=0$$

$$w_1''(1)+\mu w_2''(1)=0, \text{ with } \mu=I_2\ell_1/I_1\ell_2 \tag{4}$$

by virtue of Eqs. (3) yields the following buckling equation

$$\sin\beta+(\beta/3\mu)=0 \tag{5}$$

We can easily show that this equation has a nontrivial solution for $\mu \geq 1.53$. In this case, the nonconservatively loaded frame is a divergence-type system. Obviously, for $\mu<1.53$, the frame is a flutter-type system and its critical load can be established only by using the dynamic criterion[5,6] as is demonstrated in the next section.

Dynamic Analysis

In case of steady-state motion the following differential equations can be obtained

$$w_1''''+\beta^2 w_1''-k_1^4 w_1=0, \qquad w_2''''-k_2^4 w_2=0 \tag{6}$$

where $k_1^4=m_1\omega^2\ell_1^4/EI_1$, $k_2^4=m_2\omega^2\ell_2^4/EI_2$ (ω is the circular frequency of the steady-state motion).

Integration of Eqs. (6) and use of the support conditions yields

$$w_1(x_1)=A_1\left(\sin\bar{\lambda}x-\frac{\bar{\lambda}}{\lambda}\sinh\lambda x_1\right)+A_2(\cos\bar{\lambda}x_1-\cosh\lambda x_1)$$

$$w_2(x_2)=\bar{A}_1\sin k_2x_2+\bar{A}_3\sinh k_2x_2 \tag{7a}$$

where

$$\bar{\lambda}=[\tfrac{1}{2}\beta^2+(\tfrac{1}{4}\beta^4+k_1^4)^{1/2}]^{1/2}$$

$$\lambda=[-\tfrac{1}{2}\beta^2+(\tfrac{1}{4}\beta^4+k_1^4)^{1/2}]^{1/2}$$

$$\beta^2=\bar{\lambda}^2-\lambda^2, \qquad k_1^4=\bar{\lambda}^2\lambda^2, \qquad k_2^4=k_1^4m\rho^2/\mu,$$

$$\rho=\ell_2/\ell_1, \qquad m=m_2\ell_2/m_1\ell_1 \tag{7b}$$

Application of the boundary conditions

$$w_1'''(1)+k_1^4mw_1(1)=0, \qquad w_2(1)=0$$

$$w_1'(1)-w_2'(1)=0, \qquad w_1''(1)+\mu w_2''(1)=0 \tag{8}$$

by means of Eqs. (7a) yields the following frequency equation

$$|\alpha_{ij}|=0, \qquad i,j=1,2,3,4 \tag{9}$$

where

$$\alpha_{11}=k_1^4m\left(\sin\bar{\lambda}-\frac{\bar{\lambda}}{\lambda}\sinh\lambda\right)-\bar{\lambda}(\bar{\lambda}^2\cos\bar{\lambda}+\lambda^2\cosh\lambda)$$

$$\alpha_{12}=k_1^4m(\cos\bar{\lambda}-\cosh\lambda)+(\bar{\lambda}^3\sin\bar{\lambda}-\lambda^3\sinh\lambda)$$

$$\alpha_{13}=\alpha_{14}=0, \qquad \alpha_{21}=\alpha_{22}=0, \qquad \alpha_{23}=\sin k_2$$

$$\alpha_{24}=\sinh k_2, \qquad \alpha_{31}=\bar{\lambda}(\cos\bar{\lambda}-\cosh\lambda)$$

$$\alpha_{32}=-\bar{\lambda}\sin\bar{\lambda}-\lambda\sinh\lambda, \qquad \alpha_{33}=-k_2\cos k_2$$

$$\alpha_{34}=-k_2\cosh k_2, \qquad \alpha_{41}=\bar{\lambda}(\bar{\lambda}\sin\bar{\lambda}+\lambda\sinh\lambda)$$

$$\alpha_{42}=\bar{\lambda}^2\cos\bar{\lambda}+\lambda^2\cosh\lambda, \qquad \alpha_{43}=\mu k_2^2\sin k_2$$

$$\alpha_{44}=-\mu k_2^2\sinh k_2 \tag{10}$$

It is known[7] that the stability limit of the frame is dependent on the nature of the nondimensionalized frequency $\Omega=\lambda\bar{\lambda}$. If the nondimensionalized load β^2 is considered as a parameter, the limit of stability of the frame is determined through the relationship $\Omega=\Omega(\beta^2)$. For nonconservatively loaded systems of the divergence type (associated with non-self-adjoint differential equations), the transition from stability to instability takes place at $\Omega^2=0$. For $\beta^2 \leq \beta_{cr}^2$, all Ω are real. In contrast, for nonconservatively loaded systems of the flutter type (associated, similarly, with non-self-adjoint differential equations) stability is lost for that finite value of β^2 for which at least two consecutive frequencies become equal. Beyond this value, the frequencies become complex conjugate, and the corresponding motion is an oscillation with exponentially increasing amplitude.

Depending on the values of the parameters μ, ρ, and m, the nonconservatively loaded frame can be a divergence- or flutter-type system. Thus, Eq. (9) always has a nontrivial solution regardless of the values of the aforementioned parameters.

Nonlinear Stability Analysis

Consider an axially stressed beam subjected simultaneously to bending. On the basis of the linear curvature-displacement relationship and nonlinear axial strain-displacement relationship, the following elastic strain energy functional, in dimensionless form, can be written

$$U=\frac{1}{2}\int_0^1\left[\lambda^2\left(\xi'+\frac{1}{2}w'^2\right)^2+w''^2\right]dx \tag{11}$$

in which λ denotes the slenderness ratio of the beam.

By means of Eq. (11), application of the principle of virtual work, after performing the variations and integrating by parts, results in the following differential equations

$$\xi_i'+\tfrac{1}{2}w_i'^2=0$$

$$w_i''''-\lambda_i^2[(\xi_i'+\tfrac{1}{2}w_i'^2)w_i']'=0 \qquad i=1,2 \tag{12}$$

The respective kinematic and natural boundary conditions resulting from the variational procedure are:

$$w_1(0)=\xi_1(0)=w_1'(0)=w_2(0)=0$$

$$w_1(1)=\rho\xi_2(1), \quad \rho w_2(1)=-\xi_1(1), \quad w_1'(1)=w_2'(1) \tag{13}$$

and

$$\xi_2'(0)+\tfrac{1}{2}w_2'^2(0)=0, \qquad w_2''(0)=0$$

$$\beta^2\sin w_1'(1)-w_1'''(1)+\lambda_1^2[\xi_1'(1)+\tfrac{1}{2}w_1'^2(1)]w_1'(1)$$

$$+(\mu/\rho)\lambda_2^2[\xi_2'(1)+\tfrac{1}{2}w_2'^2(1)]=0$$

$$\rho\lambda_1^2[\xi_1'(1)+\tfrac{1}{2}w_1'^2(1)]+\rho\beta^2\cos w_1'(1)+\mu w_2'''(1)$$

$$-\mu\lambda_2^2[\xi_2'(1)+\tfrac{1}{2}w_2'^2(1)]w_2'(1)=0$$

$$w_1''(1)+\mu w_2''(1)=0 \tag{14}$$

Due to the first of Eqs. (14), Eqs. (12) and (14) can be transformed as follows

$$\xi_1'+\tfrac{1}{2}w_1'^2=-k^2/\lambda_1^2, \qquad \xi_2'+\tfrac{1}{2}w_2'^2=0$$

$$w_1''''+k^2w_1''=0, \qquad w_2''''=0 \tag{15}$$

and

$$\xi_2'(0) + \tfrac{1}{2} w_2'^2(0) = 0, \qquad w_2''(0) = 0$$

$$\beta^2 \sin w_1'(1) - k^2 w_1'(1) - w_1'''(1) = 0$$

$$\rho\beta^2 \cos w_1'(1) - k^2\rho + \mu w_2'''(1) = 0$$

$$w_1''(1) + \mu w_2''(1) = 0 \tag{16}$$

where k^2 is the nondimensionalized axial (compressive) force in the vertical member.

Integration of the last two of Eqs. (15) by means of the support conditions [Eqs. (13)] and the second of Eqs. (16) leads to

$$w_1(x_1) = A_1(\sin kx_1 - kx_1) + A_2(\cos kx_1 - 1)$$

$$w_2(x_2) = \bar{A}_1 x_2^3 + \bar{A}_2 x_2 \tag{17}$$

Subsequently, integration of the first two of Eqs. (15) and use of Eqs. (17) and the support condition $\xi_1(0) = 0$ yield

$$\xi_1(x_1) = -\frac{k^2}{\lambda_1^2}x_1 - \frac{k^2}{4}\left[A_1^2\left(3x_1 + \frac{\sin 2kx_1}{2k} - \frac{4\sin kx_1}{k}\right)\right.$$

$$\left. + A_2^2\left(x_1 - \frac{\sin 2kx_1}{2k}\right) + \frac{A_1A_2}{k}(\cos 2kx_1 - 4\cos kx_1 + 3)\right]$$

$$\xi_2(x_2) = \frac{1}{\ell} w_1(1) + \tfrac{1}{2}\left[\frac{9}{5}\bar{A}_1^2(1 - x_2^5)\right.$$

$$\left. + 2\bar{A}_1\bar{A}_2(1 - x_2^3) + \bar{A}_2^2(1 - x_2)\right] \tag{18}$$

Next, by virtue of the last of Eqs. (13) and the last of Eqs. (16), we can determine the constants of integration $\bar{A}_1$, $\bar{A}_2$ as functions of A_1, A_2, and k. Inserting subsequently their expressions into the second of the continuity conditions [Eqs. (13)] and the third and fourth of Eqs. (16), we obtain the following nonlinear equilibrium equations with respect to k, A_1, and A_2,

$$\frac{k^2}{\lambda_1^2} + \frac{k^2}{4}\left[A_1^2\left(3 + \frac{\sin 2k}{2k} - \frac{4\sin k}{k}\right) + A_2^2\left(1 - \frac{\sin 2k}{2k}\right)\right.$$

$$\left. + \frac{A_1A_2}{k}(\cos 2k - 4\cos k + 3)\right]$$

$$-\rho\left[A_1\left[k(\cos k - 1) - \frac{k^2}{3\mu}\sin k\right]\right.$$

$$\left. - A_2\left[k\sin k + \frac{k^2\cos k}{3\mu}\right]\right] = 0$$

$$\beta^2 \sin[A_1 k(\cos k - 1) - A_2 k\sin k] + k^3 A_1 = 0$$

$$\rho\beta^2\cos[A_1 k(\cos k - 1) - A_2 k \sin k]$$

$$-k^2\rho + A_1 k^2 \sin k + A_2 k^2 \cos k = 0 \tag{19}$$

The complete response of the frame, at any level of the applied load β^2 and for all values of the parameters μ, λ_1, and ρ, is known if the solution of the nonlinear equation [Eqs. (19)] is known.

In case the magnitude of the joint rotation $w_1'(1)$ up to the critical state is very small, so that we can assume $\sin w_1'(1) \simeq w_1'(1)$ and $\cos w_1'(1) \simeq 1$, the equilibrium of the frame can be established by means of the first of Eqs. (19), where the constants A_1 and A_2 are given by

$$A_1 = \frac{\rho\beta^2(k^2 - \beta^2)\sin k}{k^2[\beta^2 + (k^2 - \beta^2)\cos k]}$$

$$A_2 = \frac{\rho(k^2 - \beta^2)[k^2 + \beta^2(\cos k - 1)]}{k^2[\beta^2 + (k^2 - \beta^2)\cos k]} \tag{20}$$

Numerical Results and Discussion

Linear Stability Analysis

The frequency equation [Eq. (9)] depends on the dimensionless frequency $\Omega = \lambda\bar{\lambda} = k_1^2$, the joint stiffness μ, the length ratio ρ, the mass ratio m, and the nondimensionalized load β^2. The critical load is obtained by solving this equation on a digital computer by using a numerical scheme based on step increasing the follower compressive load β^2.

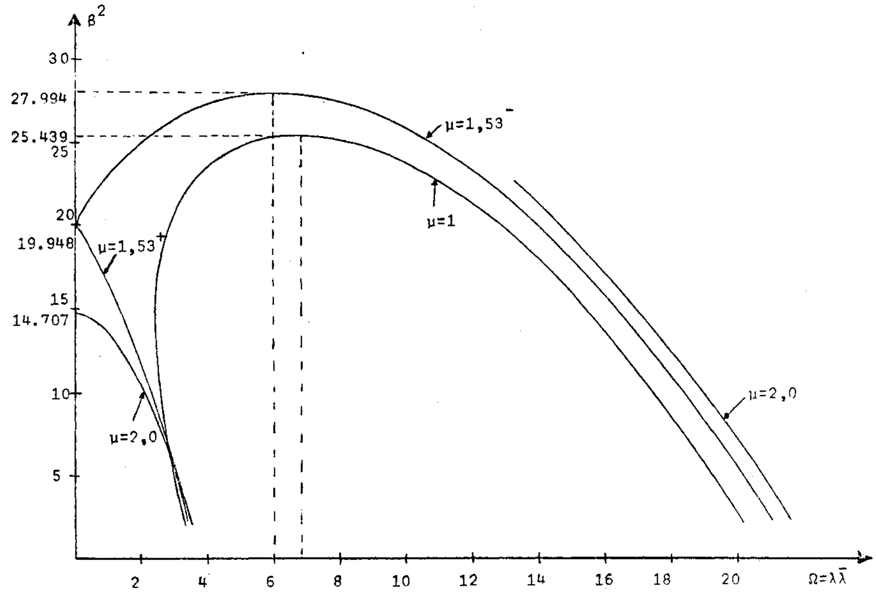

Fig. 2 Eigencurves (load β^2 vs frequency Ω) for various values of the joint stiffness μ.

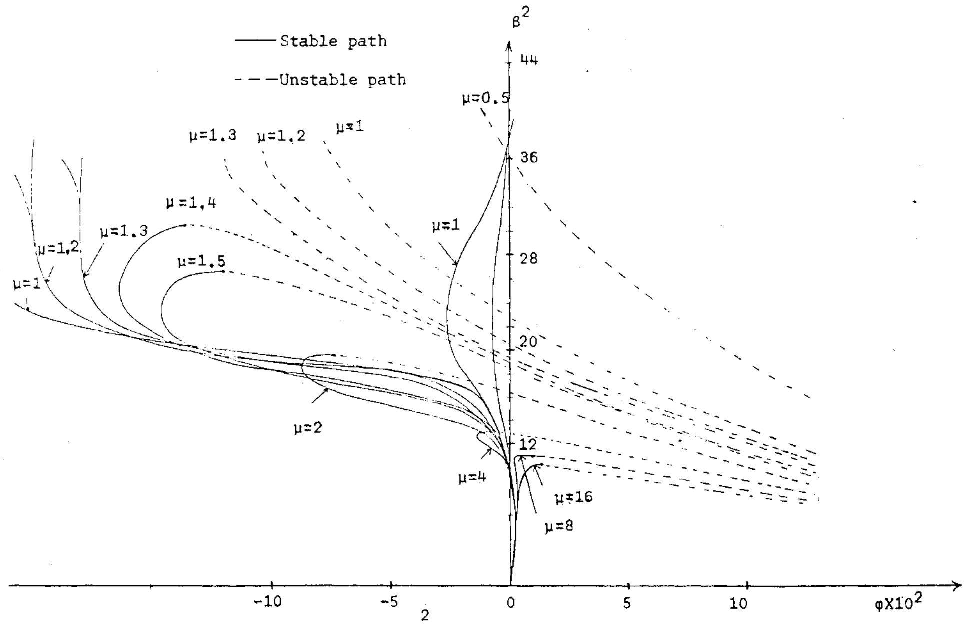

Fig. 3 Nonlinear equilibrium paths β^2 vs φ for $\lambda_l=80$, $\rho=0.25$, and $\mu=0.5$, 1, 1.2, 1.3, 1.4, 1.5, 2, 4, 8, and 16.

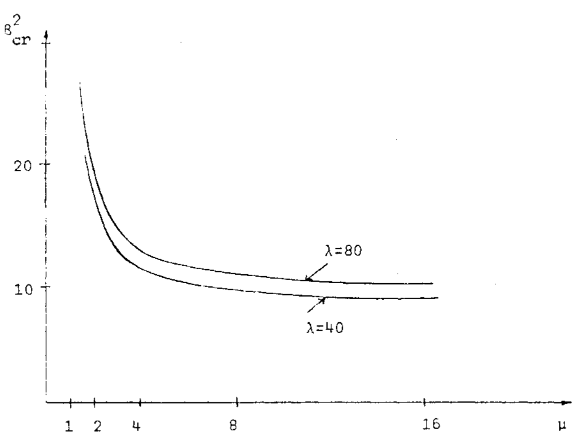

Fig. 4 External load β^2_{cr} vs joint stiffness μ for $\lambda_l=80$ and 40.

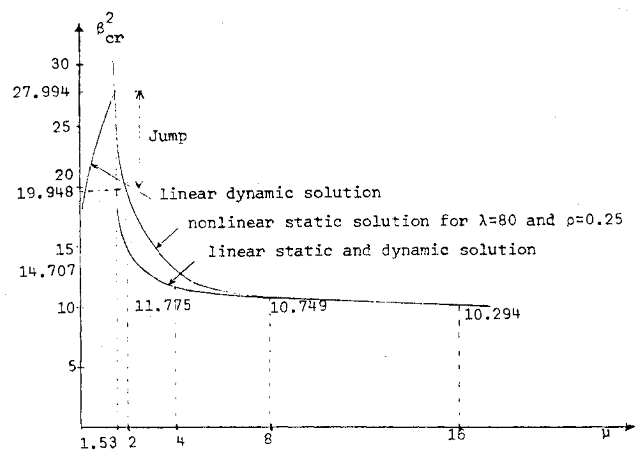

Fig. 5 Relationship between load β^2 and joint stiffness μ on the basis of linear static, linear dynamic, and nonlinear static analyses.

From the numerical solution of Eq. (9), we find that the eigencurves $\Omega=\Omega(\beta^2)$ for $\mu>1.53$ intersect the β^2 axis at points which depend only on μ and are independent of the parameters ρ and m. For that range of values of stiffness μ, the instability is of the divergence type. This result is in agreement with the static buckling equation [Eq. (5)] which yields the same divergence critical loads. For $\mu<1.53$, the coincidence of the two first eigenmodes takes place at $\Omega^2\neq 0$ which means that the frame behaves as a flutter-type system. In this case the static buckling equation admits only the trivial solution $\beta^2=0$.

The eigencurves corresponding to the first two eigenmodes for various values of the stiffness parameter μ are shown in Fig. 2. At the critical value $\mu=1.53$, the buckling mechanism changes from divergence to flutter instability and vice versa. Moreover, this change is associated with a considerable discontinuity[7] in the critical load. Specifically, at $\mu=1.53^+$ the divergence critical load is equal to 19.948, whereas at $\mu=1.53^-$ the flutter (critical) load becomes equal to 27.994. Another kind of jump in the flutter load has been presented in Ref. 8.

In conclusion the linear static stability analysis can be applied for $\mu>1.53$, whereas the linear dynamic analysis is valid for the entire range of values of μ. Thus, the region of flutter-type instability is very limited and the divergence-type instability predominates.[7,9] However, the foregoing linear stability analyses do not give any information concerning the magnitudes of displacements at the critical state. Subsequently, some light on this question and thus the validity of these analyses will be shown by using a nonlinear static stability analysis.

Nonlinear Stability Analysis

Solving numerically the system of nonlinear equations [Eqs. (19)] for given values of the parameters μ, λ_l, and ρ and each level of the load β^2, we can establish all of the equilibrium paths as plots of the load β^2 vs some characteristic displacement, viz., $w_l(1)$, $\xi_l(1)$, or $w_l'(1)$. In all cases the critical load is determined as a limit point load.

From Fig. 3 we can see the relationship between the joint rotation $\varphi=w_l'(1)$ and the nondimensionalized β^2 for a two-bar frame with $\rho=0.25$, $\lambda_l=80$, and various values of the stiffness parameter μ, viz., $\mu=16$, 8, 4, 2, 1.5, 1.4, 1.3, 1.2, and 1.0. It is worth observing that for $\mu=8$ and 16 the rotation φ_{cr} corresponding to the critical (limit) point load is positive, whereas for $\mu<4$, φ_{cr} becomes negative. Certainly, there is some intermediate value of μ between 4 and 8 for which $\varphi_{cr}=0$. The decrease of the parameter μ implies a considerable reduction to φ_{cr}; moreover for some critical value of μ between 1.4 and 1.3, φ_{cr} becomes infinitely large (the corresponding limit point goes to infinity). Beyond the value of $\mu=1.5$, the validity of these results is questionable for the magnitudes of the joint displacements are not consistent with the nonlinear theory employed herein. However, we can observe that as μ approaches some critical value between $\mu = 1.4$ and 1.3, the equilibrium path degenerates into several curves which are physically unacceptable solutions. Consequently, for $\mu < \mu_{cr}$ the application of the nonlinear static analysis fails and the foregoing region of flutter-type instability cannot be discussed. Nevertheless, the use of the nonlinear static analysis leads to two very important observations. The divergence-type instability is associated with a limit point instability, and the foregoing jump indicates that the magnitude of the corresponding displacements are infinitely large.

From Fig. 4 we can see the effect of joint stiffness μ on the critical load for $\lambda_l = 40$ and 80. Clearly as λ_l decreases, the load-carrying capacity also decreases. This result is also valid for the conservatively loaded frame presented in Ref. 10.

Comparison between Linear and Nonlinear Solutions

From Fig. 5 we can see the dependence of the critical load β^2 on the stiffness parameter μ on the basis of the linear static and dynamic analysis as well as of the nonlinear analysis with $\lambda_l = 80$ and $\rho = 0.25$. Clearly, the linear static and linear dynamic critical (divergence) loads coincide for $\mu > 1.53$. The corresponding nonlinear critical loads are practically the same as the linear critical loads for $\mu > 6$, whereas for $1.53 < \mu < 6$ they become substantially higher than the last ones. The difference between the nonlinear and linear critical load at $\mu = 1.53$ is approximately equal to the aforementioned discontinuity in the critical load. The validity of both the nonlinear and linear analyses is questionable for μ approaching its critical value, $\mu_{cr} = 1.53$. This fact is due to the large magnitudes of the corresponding critical state displacements which are not consistent with the limitations of the linear stability theory. We also emphasize that all of the critical states for $\mu > 1.53$ are unstable for, as shown previously, the frame loses its stability through a limit point (snap-through) buckling.

The accuracy of the flutter load obtained by means of dynamic analysis (for $\mu > 1.53$) can be discussed only by using a nonlinear dynamic stability analysis, because both static (linear and nonlinear) analyses fail at that range of values of μ.

From this investigation and that presented in Ref. 4, we can conclude that the types of critical points appearing in conservative systems appear also in nonconservative divergence type systems.

Conclusions

In this investigation the elastic stability of a nonconservatively loaded frame subjected at its joint to a tangential load is discussed on the basis of linear static, linear dynamic, and nonlinear static analyses. The following constitute the most important conclusions:

1) Both types of instability, viz., divergence and flutter, can occur depending on the amount of a joint stiffness parameter.

2) Linear and nonlinear static methods of stability analysis cannot be employed for investigating the stability of flutter-type systems. In this case dynamic (linear or nonlinear) stability analysis must be employed.

3) The region of flutter-type instability is very limited and therefore the divergence-type instability predominates. However, a nonlinear stability analysis is needed for determining the type of the critical states of equilibrium.

4) By using the nonlinear analysis we show that the frame loses its stability through divergence which is associated with a limit point instability. Consequently, the stability of the frame is governed by snap-through buckling.

5) The jump phenomenon in the critical load occurring when the buckling mechanism changes from flutter to divergence and vice versa is a consequence of the application of the linear methods of stability analysis.

6) From the nonlinear stability analysis it is established that the displacements corresponding to that jump become extremely large, and therefore the results obtained by the linear (static or dynamic) method of analysis are not consistent with its limitations.

7) For large values of the joint stiffness parameter μ, the linear and nonlinear static analyses give practically identical results, whereas for small values of this parameter an appreciable disagreement between the linear and nonlinear critical load is observed. This disagreement becomes extremely pronounced as the foregoing parameter approaches its critical value which corresponds to the jump phenomenon.

8) The types of critical points of conservatively loaded systems are also valid for nonconservatively loaded systems of the divergence type.

References

[1] Burgess, W. and Levinson, M., "The Post-Flutter Oscillations of Discrete Symmetric Structural Systems with Circulatory Loading," *International Journal of Mechanical Science,* Vol. 14, Oct. 1972, pp. 471-488.

[2] Plaut, R. H., "Post-Buckling Behavior of Continuous Non-Conservative Elastic Systems," *Acta Mechanica,* Vol. 30, Jan. 1978, pp. 51-64.

[3] Kounadis, A. N., Giri, J., and Simitses, G. J., "Divergence Buckling of a Simple Frame Subject to a Follower Force," *Transactions of ASME, Journal of Applied Mechanics,* Vol. 45, June 1978, pp. 426-428.

[4] Kounadis, A. N., "The Effects of Some Parameters on the Nonlinear Divergence Buckling of a Nonconservative Simple Frame," *Journal de Mēcanique Appliqueé,* Vol. III, April 1979.

[5] Kounadis, A. N., "Stability of Elastically Restrained Timoshenko Cantilevers with Attached Masses Subjected to a Follower Force," *Transactions of ASME, Journal of Applied Mechanics,* Vol. 44, Dec. 1977, pp. 731-736.

[6] Kounadis, A. N. and Katsikadelis, J. T., "Shear and Rotatory Inertia Effect on Beck's Column," *Journal of Sound and Vibration,* Vol. 49, Nov. 1976, pp.171-178.

[7] Kounadis, A. N. and Economou, A. P., "The Effects of the Joint Stiffness and of the Constraints on the Type of Instability of a Frame under a Follower Force," *Acta Mechanica,* Vol. 36, March 1980, pp. 157-168.

[8] Kounadis, A. N. and Katsikadelis, J. T., "On the Discontinuity of the Flutter Load for Various Types of Cantilevers," *International Journal of Solids and Structures,* Vol. 16, April 1980, pp. 375-383.

[9] Kounadis, A. N., "On the Static Stability Analysis of Elastically Restrained Structures under Follower Forces," *AIAA Journal,* Vol. 18, April 1980, pp. 473-476.

[10] Kounadis, A. N., Giri, J., and Simitses, G. J., "Nonlinear Stability Analysis of an Eccentrically Loaded Two-Bar Frame," *Transactions of ASME, Journal of Applied Mechanics,* Vol. 44, Dec. 1977, pp. 701-706.